Generations of Higgs Bosons in Supersymmetric Models

Alfredo Aranda* and Marc Sher†

Nuclear and Particle Theory Group, Department of Physics, College of William and Mary,

Williamsburg, VA 23187-8795

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Abstract

We examine extensions of the MSSM with more than one generation of Higgs bosons. If one assumes that a symmetry eliminates the tree-level FCNC, then the extra scalar bosons do not acquire VEVs, do not couple to fermions and do not mix with the ordinary Higgs bosons; the lightest is absolutely stable. The mass splittings between the two lightest neutral scalars, ϕ_S, and ϕ_P, and between those and the lightest charged scalar, ϕ_+, are calculated. For most of the parameter space, the latter is $1.5-6.0$ GeV heavier than the ϕ_S, and the ϕ_P is $200-1500$ MeV heavier. The signatures are quite dramatic. The ϕ_+ will decay at the vertex; the signature for this decay will be like that of a chargino with a nearly degenerate undetected neutralino. Roughly half of the time, the decay is into a ϕ_P, which subsequently decays into ϕ_S and a π^0, ρ or lepton pair at a macroscopic distance (millimeters to tens of centimeters) from the vertex. Finally, the possibility that the symmetry that eliminates FCNC is a flavor symmetry is discussed. In an example, the U(2) model, tree-level FCNC processes can be calculated in terms of quark masses. The strongest constraint on this model is from $D-\bar{D}$ mixing, which should be within an order of magnitude of the current bound.

*fefo@physics.wm.edu

†sher@physics.wm.edu

I. INTRODUCTION

One of the great mysteries of the standard model is the number of fermion generations. Nothing in the structure of the standard model gives any clue as to what this number should be, and the same is true of the most popular extensions of the standard model − grand unified theories and the MSSM. The only information we have about the number of fermion generations is phenomenological. It is interesting that this situation is the same for the Higgs bosons, i.e., there is no clue as to the number of generations of Higgs bosons (in supersymmetric models each generation consisting of two doublets of opposite hypercharge). One might *a priori* expect the number of Higgs generations to be equal to the number of fermion generations. This is because in some supersymmetric grand unified theories, such as E_6, the Higgs bosons and fermions belong to the same representations. Just as in the fermionic sector, the number of Higgs generations can, at present, only be determined phenomenologically.

Of course, the number of Higgs generations detected experimentally at present is zero. However, a strong clue comes from the absence of tree-level flavor-changing neutral currents (FCNC). In the standard model with a single Higgs doublet, the Yukawa coupling matrices are proportional to the fermionic mass matrices. Diagonalizing the latter thus automatically diagonalizes the former, eliminating FCNC. If one or more Higgs doublets are added to the standard model, diagonalizing the mass matrices does not in general diagonalize the Yukawa coupling matrices. In minimal supersymmetric extensions of the standard model, there are two Higgs doublets of opposite hypercharge, one couples to the $Q = 2/3$ quarks while the other couples to the $Q = 1/3$ quarks, thereby eliminating FCNC. Again, with an extra pair of doublets added to these minimal models, FCNC naturally emerge.

Since we are interested in the possibility of having additional generations of Higgs bosons, it is important to solve the problem of FCNC [1]. There are two approaches [1], the first one [2] eliminates them completely by imposing a discrete symmetry. The second approach [3] does not eliminate FCNC, but makes them sufficiently small by assuming that the flavor-changing neutral couplings of quarks q_i and q_j are proportional to the geometric mean of the Yukawa couplings of the same quarks. In this case, the most dangerous FCNC, which involve the down and strange quarks, are suppressed by the small down and strange Yukawa couplings.

In Ref. [4], the first approach (applying a discrete symmetry to remove FCNC) was applied to the possibility of adding extra generations of Higgs bosons to the MSSM. They considered three generations of Higgs bosons: six doublets, three of each hypercharge. It was shown that if one assumes that some symmetry (discrete, continuous, global or local) suppresses FCNC, then the extra generations, which they called "pseudoHiggs bosons" decouple completely from the "standard" Higgs doublets. These extra fields do not acquire vacuum expectation values, do not mix with the "standard" doublets and do not couple to fermions. The lightest of these extra scalars is stable, and much of Ref. [4] was devoted to the possibility that this scalar could constitute the dark matter.

It was also shown in Ref. [4] that the mass matrices of the extra scalars have a very

[1]Although generally discussed in the context of two-generation models, these approaches apply more generally.

unusual form. In spite of the fact that virtually all of the elements of these matrices were arbitrary and unknown parameters, it was shown that the lightest neutral scalar and the lightest neutral pseudoscalar were degenerate in mass at tree level, and that the lightest charged scalar was a few GeV heavier. There was, however, no detailed calculation of the mass splitting due to radiative corrections, and no detailed analysis of the phenomenological signatures (except in the context of the Z width, should the masses be below 45 GeV).

In this article, we analyze the MSSM with extra generations of Higgs doublets. Initially, we assume that there is a symmetry which eliminates tree-level FCNC (at this point, we do not worry about the precise nature of the symmetry). In Section II, we present the model and show the surprising mass degeneracy of the lightest pseudoHiggs bosons. In Section III, we calculate the mass splitting and look at the decay modes of the lightest pseudoscalar and charged scalar. In Section IV the phenomenology is discussed, and it is shown that standard searches for heavy leptons with neutrinos a few GeV lighter will be sensitive to these particles, but an additional signature will be present in over half of the events. In Section V we relax the assumption that tree-level FCNC are eliminated, and consider a simple flavor symmetry, based on U(2), and show how the resulting tree-level FCNC can be calculated in terms of fermion masses.

II. THE MODEL

Let us first summarize the model discussed in Ref. [4]. We consider the supersymmetric standard model with three generations of Higgs doublets. The most general superpotential is given by

$$W = \mu_{ij} H_i \bar{H}_j + f_{ijk} Q_i U_j H_k + g_{ijk} Q_i D_j \bar{H}_k + h_{ijk} L_i E_j \bar{H}_k \,, \tag{2.1}$$

where the lowercase Latin indices run over 1,2,3. It is assumed that some symmetry eliminates the tree-level FCNC in the quark and lepton sectors. Given this assumption, a basis can be chosen in which only one generation, conventionally chosen to be H_3 and $\bar{H}_3$, couples to quarks and leptons. This means that H_3 and $\bar{H}_3$ must have different quantum numbers under that symmetry than the other $H_{1,2}$ and $\bar{H}_{1,2}$, in order that only the former couple to fermions.

The most general soft SUSY-breaking terms involving only the Higgs fields are

$$W_{soft} \supset m^2_{H_{ij}} H_i^\dagger H_j + m^2_{\bar{H}_{ij}} \bar{H}_i^\dagger \bar{H}_j - B\mu_{ij} \left(H_i \bar{H}_j + h.c. \right) . \tag{2.2}$$

Since H_3 and $\bar{H}_3$ have different quantum numbers than the other $H_{1,2}$ and $\bar{H}_{1,2}$ under the symmetry, then the quadratic terms involving only one of them vanish, i.e. $m^2_{H_{i3}} = m^2_{\bar{H}_{i3}} = m^2_{H_{3i}} = m^2_{\bar{H}_{3i}} = \mu_{i3} = \mu_{3i} = 0$ for $i = 1, 2$. Thus there are no quadratic terms mixing the third generation of Higgs fields with the other two. In addition, Ref. [4] shows that equality of scalar masses at the unification scale automatically implies that the first and second generation fields do not get vacuum expectation values. Note that we have four new scalar fields, four new pseudoscalar and four new pairs of charged scalars. In the following, we specialize to the case in which there is only a single generation of extra Higgs fields, denoted by H_X and $\bar{H}_X$. This is primarily for simplicity−including the additional generations does not affect our results. In fact, if the two additional generations have different quantum numbers under the symmetry that eliminates FCNC, then the generations will decouple and this specialization is completely general. Even if the coupling between the two generations exists, however, the results we present below are completely unaffected—the results would then simply apply to the lightest of the scalar fields.

So, assuming only one extra generation of Higgs fields, the scalar potential can be written as

$$\begin{aligned} V &= m_X^2|H_X|^2 + \bar{m}_X^2|\bar{H}_X|^2 + m_1^2|H|^2 + m_2^2|\bar{H}|^2 + \left(\mu_X H_X \bar{H}_X + \mu H \bar{H} + h.c.\right) \\ &+ \frac{g^2}{8}\sum_a \left|H_X^\dagger \tau_a H_X + \bar{H}_X^\dagger \tau_a \bar{H}_X + H^\dagger \tau_a H + \bar{H}^\dagger \tau_a \bar{H}\right|^2 \\ &+ \frac{g'^2}{8}\left||H|^2 + |H_X|^2 - |\bar{H}|^2 - |\bar{H}_X|^2\right|^2 , \end{aligned} \tag{2.3}$$

where H and $\bar{H}$ are the standard MSSM doublets, τ_a are the Pauli matrices, and where μ and μ_X are arbitrary parameters of dimension $(\mathrm{GeV})^2$. The full Lagrangian has a symmetry under which the H_X fields change sign (it is actually a global U(1) symmetry), and thus the lightest of these fields is stable.

The mass matrices of the scalars can be calculated. The mass matrix for the additional neutral scalar fields is given by

$$M_S^2 = \begin{pmatrix} m_X^2 - \frac{1}{2}M_Z^2\cos 2\beta & -\mu_X \\ -\mu_X & \bar{m}_X^2 + \frac{1}{2}M_Z^2\cos 2\beta \end{pmatrix} , \tag{2.4}$$

and the matrix for the additional neutral pseudoscalars is given by

$$M_P^2 = \begin{pmatrix} m_X^2 - \frac{1}{2}M_Z^2\cos 2\beta & \mu_X \\ \mu_X & \bar{m}_X^2 + \frac{1}{2}M_Z^2\cos 2\beta \end{pmatrix} . \tag{2.5}$$

The mass matrix for the charged scalars is

$$M_+^2 = \begin{pmatrix} m_X^2 - \frac{1}{2}M_Z^2\cos 2\bar{\beta} & -\mu_X \\ -\mu_X & \bar{m}_X^2 + \frac{1}{2}M_Z^2\cos 2\bar{\beta} \end{pmatrix} , \tag{2.6}$$

where $\cos 2\bar{\beta} \equiv \cos 2\beta \cos 2\theta_W$. Note that the mass matrices for the scalar and pseudoscalar are *identical* except for the sign of the even-odd elements (this is true even in the case of many additional generations all coupled together). As a result, the secular equation is identical for both mass matrices, and so the eigenvalues are the same. However, due to the sign difference, we will see that the degeneracy is lifted by radiative corrections. As shown in Ref. [4], the coupling of the scalar and pseudoscalar to the Z is completely independent of any mixing angles in the Higgs sector, and is thus determined.

The charged scalar mass matrix is identical to the neutral scalar mass matrix with $M_Z^2 \to M_Z^2 \cos 2\theta_W$. This results in a *larger* mass for the lightest charged scalar, but only slightly larger—we will see that a few GeV is a typical size.

None of this is new. It was discussed in much more detail in Ref. [4]. However, they did not explicitly calculate the splitting of the degeneracy between the neutral scalars. They just argued that it is approximately a GeV. Since the leptonic decay of the heavier of the two will vary as the fifth power of the mass difference, a more precise calculation is needed to explore the phenomenology. They also did not explore the difference between the charged and neutral scalar masses, which is critical to detection, nor did they look at the decay modes and signatures. We now turn to these issues.

III. MASS SPLITTINGS AND DECAY MODES

The mass matrices Eq. (2.4),(2.5), and (2.6) have four unknown parameters: m_X^2, $\bar{m}_X^2$, μ_X, and $\tan\beta$. However, as noted above, the beta functions for m_X^2 and $\bar{m}_X^2$ are identical, and equality of the masses at a high scale implies that they are equal at all scales. We thus set them equal to each other, and briefly discuss the effect of relaxing this assumption later. We also see that the results are extremely insensitive to $\tan\beta$. Thus, there are effectively two parameters. These two parameters give the masses of the two neutral scalars, the two neutral pseudoscalars and the two charged scalar pairs, as well as all of the mixing angles. The coupling to the vector bosons are thus determined in terms of these parameters, and are given in Ref. [4].

We are most interested in the splitting between the masses of the lightest neutral scalar, ϕ_S, and the neutral pseudoscalar, ϕ_P. This splitting comes from the self-energy diagrams in which the ϕ_S or the ϕ_P goes into a charged scalar, ϕ_+, and a $W-$boson. The couplings of the ϕ_S or ϕ_P to the ϕ_+ and W are proportional to $\cos(\theta_+ - \theta_S)$ and $\cos(\theta_+ - \theta_P)$, respectively. Since the above mass matrices have $\theta_S = -\theta_P$, these couplings are different, lifting the mass degeneracy. We include contributions from both charged scalars. The mass splitting is given, with $\delta M = M_P - M_S$, by

$$2\sin^2\theta_S \left[f\left(\frac{\bar{m}_+^2}{M_S^2}, \frac{M_W^2}{M_S^2}\right) - f\left(\frac{M_+^2}{M_S^2}, \frac{M_W^2}{M_S^2}\right)\right] , \tag{3.1}$$

where

$$\begin{aligned} f(a,b) \equiv \frac{g^2}{32\pi^2} \Bigg(\int_0^1 & dx(3x^2 - 6x + 4 + ax + b(1-x)) \\ & \times \ln(ax + b(1-x) - x(1-x)) - \frac{3a + 3b - 1}{6} \Bigg) , \end{aligned} \tag{3.2}$$

where $\bar{m}_+$ is the mass of the heavier charged scalar, and we have used the fact [4] that θ_+ is very close to θ_S. This result is plotted as a function of M_S and μ_X in Figure 1. We have plotted the results for $\tan\beta = 2$, but the results only change slightly for different $\tan\beta$ (for $\tan\beta = \infty$, the splitting changes by $20 - 30\%$, thus slightly changing the contour lines). We see that the pseudoscalar is the heavier of the two, and for much of the parameter space is within a factor of 2 of 500 MeV. The region in which the splitting is below 200 MeV (which would give a decay length so long that the heavier would escape a detector before decaying) is very small, as is the region in which the splitting is above 2000 MeV (which would give a decay at the vertex). Note that we have not included the contributions from the supersymmetric partners of the ϕ_+ and the W. These contributions will depend on additional parameters (chargino mixing, for example). Obviously, in the limit in which the SUSY breaking scale is fairly large, these contributions will be small, but in general they could be sizeable.

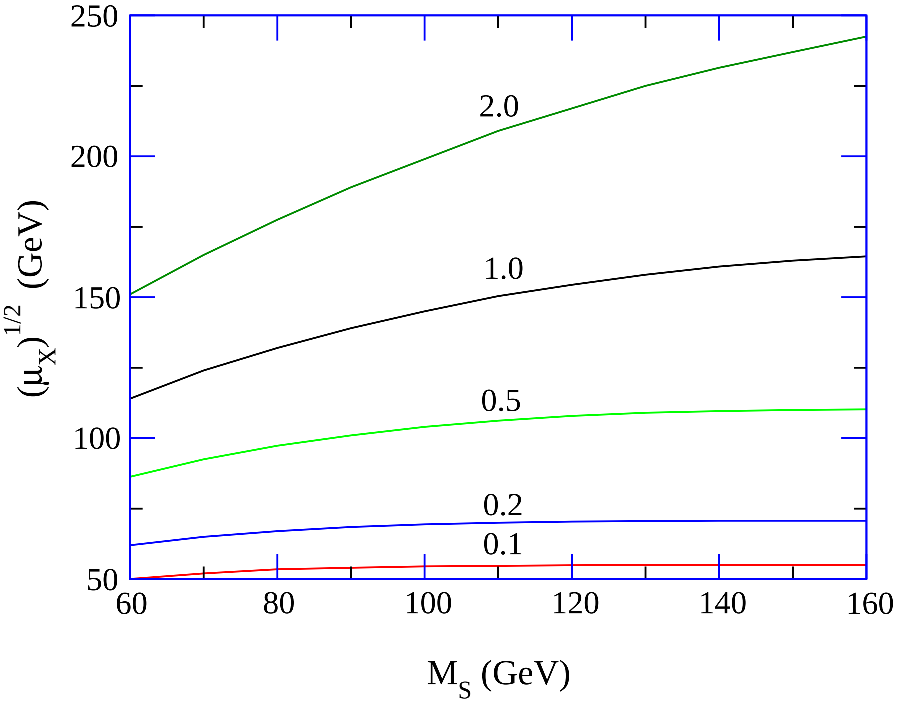

FIG. 1. The mass splitting, in GeV, between the neutral pseudoscalar and the neutral scalar, as a function of the scalar mass and the parameter μ_X.

Note, however, that the parameter μ_X in Figure 1 is completely undetermined (and possibly not determinable in the foreseeable future); our purpose in showing the dependence is simply to demonstrate the values of the splitting as a function of the size of the parameter space. This will not be changed by inclusion of supersymmetric contributions. Should the assumption $m_X = \bar{m}_X$ be relaxed, the results also do not change substantially.

We now turn to the mass difference between the lightest charged Higgs and ϕ_S. Here, the largest contribution to the mass difference appears at the tree level, however we have also included radiative corrections (which include $W-\phi_S$, $W-\phi_P$, $Z-\phi_+$, $\gamma-\phi_+$ corrections for the charged Higgs and the above corrections plus $Z-\phi_P$ for the ϕ_S) in the mass difference.

The resulting splittings are shown in Figure 2. We see that the splitting varies from 1.5 to 6.0 GeV over most of the parameter space. This implies that the ϕ_+ will decay at the vertex. Also, this mass range is detectable at LEP$-$ searches for heavy leptons with a nearly degenerate neutrino can detect mass splittings down to approximately 1.5 GeV.

The most interesting decay, of course, is that of ϕ_P, which, due to the small mass splitting with ϕ_S, will have a substantial decay length. What are the decay modes? The ϕ_P decays into a ϕ_S and a virtual Z. The primary decay modes are $\phi_S\pi^0$, $\phi_S e^+e^-$, $\phi_S\mu^+\mu^-$, and $\phi_S\rho^0$. There are also multihadronic decays, if the splitting is large enough. We do not calculate those, but will discuss them at the end.

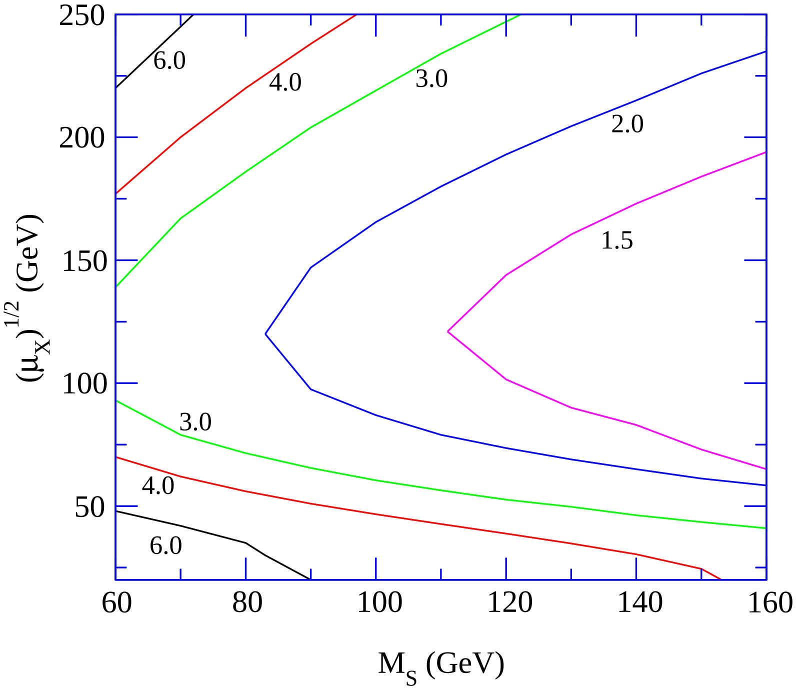

FIG. 2. The mass splitting, in GeV, between the lightest charged scalar and the lightest neutral scalar, as a function of the latter's mass and the parameter μ_X.

The decay widths can be calculated. The $\phi_S\pi^0$ decay is calculated using standard chiral perturbation theory techniques, the $\phi_S\rho^0$ is calculated using vector dominance, and the leptonic decays are calculated in the standard manner. We find that

$$\Gamma(\phi_S\pi^0) = \frac{G_F^2 f_\pi^2 (\delta M)^3}{4\pi}\left(1 - \frac{m_\pi^2}{(\delta M)^2}\right)^{1/2}, \tag{3.3}$$

$$\Gamma(\phi_S\rho^0) = \frac{G_F^2 f_\rho^2 (\delta M)^3}{32\pi^2}\left(1 - \frac{m_\rho^2}{(\delta M)^2}\right)^{3/2}, \tag{3.4}$$

and where the $\phi_S e^+e^-$ and the $\phi_S\mu^+\mu^-$ decay widths have been calculated numerically (they are proportional to $(\delta M)^5$).

These results are plotted in Figure 3. We see that for much of the parameter space the $\phi_S\pi^0$ final state is the dominant decay, with the other modes becoming substantial once the splitting exceeds 1000 MeV. Note that $\Gamma = 10^{-14}$ GeV corresponds to a decay distance $c\tau$ of two centimeters, and thus we see that for the bulk of parameter space, the decay length is between two millimeters and twenty centimeters. This length is obviously of great phenomenological interest. We now turn to the experimental detection of these extra scalars.

IV. EXPERIMENTAL CONSIDERATIONS

The spectrum of the lightest additional scalars consists of a charged scalar, ϕ^+, and two neutral scalars, ϕ_P and ϕ_S. The lightest scalar is ϕ_S, which is stable and weakly interacting; it will appear as a heavy neutrino and will escape any detector. The second lightest scalar is the ϕ_P, which has the partial decay widths given in Figure 3. The heaviest is the ϕ^+, which is $2-6$ GeV (for most of parameter space) heavier than the ϕ_S. Thus, the ϕ^+ will decay at the vertex into a virtual W and either a ϕ_P or a ϕ_S. Let us first look at experimental detection of these particles qualitatively, and then more quantitatively.

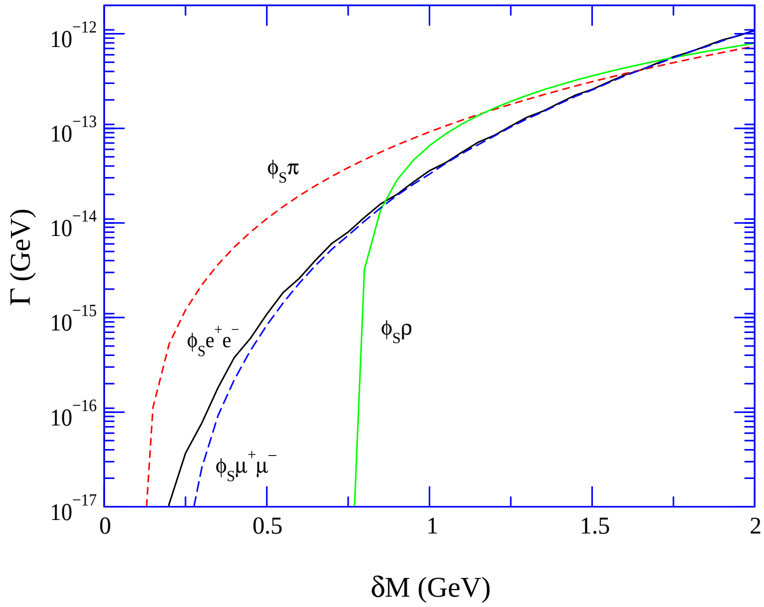

FIG. 3. Decay modes of the pseudoscalar, ϕ_P, as a function of the mass splitting.

From an experimental point of view, the ϕ^+ looks like a heavy, charged lepton with an associated neutrino which is a few GeV lighter. Searches for heavy leptons with nearly degenerate neutrinos will be sensitive to this particle. (The fact that the ϕ's are scalars will appear in the angular distribution in the production rate, as we will see). Recently, DELPHI [5] has conducted a search for charginos which are nearly mass-degenerate with the lightest neutralino. The signatures here will be very similar. They show that the standard chargino searches will cover mass splittings down to about 3 GeV, and that initial state radiation can extend this reach. Thus it is likely that LEP experiments will be able to cover much of the allowed parameter space in this model if the extra scalars are light enough. As we will see shortly, the fact that the model has so few parameters allows a precise determination of the production cross-section, unlike the chargino case, which depends on various mixing angles, scalar neutrino masses, etc.

Once the ϕ^+ decays, it will decay into either a ϕ_S, which disappears like a heavy neutrino or neutralino, or a ϕ_P, which will have a decay length ranging from two millimeters to twenty centimeters over much of parameter-space. We will see that the latter decay happens at a substantial rate, and thus one expects many of the "chargino" events to have an additional vertex away from the interaction region. This could allow for very dramatic signatures.

More quantitatively, the production cross-section for $\phi^+\phi^-$ at an e^+e^- collider is given by

$$\sigma = \frac{\pi\alpha^2}{3s}\left(1-\frac{4M_+^2}{s}\right)^{3/2}\left[1+\frac{A}{(1-\frac{M_Z^2}{s})^2}\right] \tag{4.1}$$

where $A \equiv \frac{\cos^2 2\theta_W}{4\sin^4 2\theta_W} \sim 0.15$, and we have not included the vector coupling of the electron to the Z, which is proportional to $\frac{1}{4} - \sin^2\theta_W$. For $\sqrt{s} \sim 200$ GeV, this gives a cross-section of 0.6 picobarns times the phase space factor. The angular distribution is the usual $\sin^2\theta$ distribution for scalar particles.

Once produced, the ϕ^+ will decay at the vertex into a W^+ and a ϕ_S or a ϕ_P. The ratio of ϕ_P's produced to ϕ_S's produced is given by $\sec^2 2\theta_S$ times the phase space factor of $\left(\frac{M_+ - M_P}{M_+ - M_S}\right)^5$, where θ_S is the angle that diagonalizes the scalar mass matrix [4]

$$\cos^2 2\theta_S = \frac{M_Z^4 \cos^2 2\beta}{M_Z^4 \cos^2 2\beta + 4\mu_X^2} \tag{4.2}$$

For $\tan\beta = 2$,this gives $\sec^2 2\theta_S \sim 17(\mu_X^{1/2}/100\mathrm{GeV})^4$. This result is fairly insensitive to $\tan\beta$. From Figures 1 and 2, one can read off the final ratio of ϕ_P to ϕ_S production. For much of parameter-space, the ratio is greater than unity, so that the decay of the ϕ_+ into ϕ_P will occur over half the time.

When a ϕ_P is produced, it will travel a macroscopic distance before decaying into either one of the final states shown in Figure 3 or into a multi-hadronic final state. Based on τ decays, we do not expect multi-hadronic final states to be dominant.

Thus, the detection is similar to the detection of a chargino (heavy lepton) with a nearly mass-degenerate neutralino (neutrino), however a large fraction of the events will be accompanied by a soft π^o, ρ^o or lepton pair emerging a macroscopic distance from the vertex. Both processes, detection of the ϕ^+ decay and detection of the ϕ_P decay, have significant backgrounds, but the simultaneous detection is likely to be much simpler.

V. U(2) MODEL

In this section we consider relaxing the assumption that a symmetry forbids tree level FCNC. We do this by giving the model an explicit flavor symmetry. A very successful and elegant flavor symmetry that has been considered in the literature is the (horizontal) U(2) flavor symmetry [6]. In this model, the matter fields of the first and second generations transform as the components of a doublet, while the third generation transforms as a singlet, i.e. if we denote the matter fields by ψ, then $\psi = \psi_a + \psi_3$, where $a = 1, 2$. The Higgs doublets transform as singlets. The minimal model contains three flavon fields which are responsible, through their vacuum expectation values (vevs), for the breaking of the flavor symmetry. The flavons consist of a doublet ϕ, a triplet S, and a (antisymmetric) singlet A. The breaking occurs in two steps

$$U(2) \xrightarrow{\epsilon} U(1) \xrightarrow{\epsilon'} \ nothing\,, \tag{5.1}$$

where ϵ is the vev of ϕ and S, while ϵ' is the vev of A. We will consider the unified version [6] in which the model is embedded in a SU(5) GUT. In this case, the flavon fields also transform under SU(5), and a new flavon Σ is introduced. The flavons and their transformations are

$$\phi \sim (\mathbf{1},\mathbf{2})\ , S \sim (\mathbf{75},\mathbf{3})\ , \tag{5.2}$$

$$A \sim (\mathbf{1},\mathbf{1})\ , \Sigma \sim (\mathbf{24},\mathbf{1})\,, \tag{5.3}$$

where the numbers in parenthesis correspond to the transformations under SU(5) and U(2) respectively. They acquire the following vevs:

$$\langle S\rangle \approx \begin{pmatrix} 0 & 0 \\ 0 & \epsilon \end{pmatrix} \,, \quad \langle\phi\rangle \approx \begin{pmatrix} 0 \\ \epsilon \end{pmatrix} \,,$$

$$\langle A\rangle \approx \; \epsilon^{ab}\, \epsilon' \quad , \quad \langle\Sigma\rangle \approx \epsilon \,. \tag{5.4}$$

This model successfully reproduces the observed quark mass ratios and CKM angles, as well as the lepton mass ratios [6]. Let's now consider the possibility of having three Higgs generations and letting them transform non-trivially under the flavor symmetry [7]. The matter fields are in $\bar{\mathbf{5}}$ ($\bar{F}$), and $\bar{\mathbf{10}}$'s (T) of SU(5). Then, their transformation properties are $(\mathbf{10}, \mathbf{2} \oplus \mathbf{1})$ and $(\bar{\mathbf{5}}, \mathbf{2} \oplus \mathbf{1})$ respectively, where again the first term in the parenthesis corresponds to SU(5) and the second to U(2). There are six Higgs doublets with components H and $\bar{H}$ transforming as $(\mathbf{5}, \mathbf{2} \oplus \mathbf{1})$ and $(\bar{\mathbf{5}}, \mathbf{2} \oplus \mathbf{1})$ respectively.

The Yukawa part of the superpotential is

$$\begin{aligned} W_Y = {} & T_3 H_3 T_3 + T_3 \bar{H}_3 \bar{F}_3\, \xi + \frac{1}{M_f} \Big[T_3 H_a \phi^a T_3 + T_3 \phi^a H_3 T_a + \Big(T_3 \phi^a \bar{H}_3 \bar{F}_a + T_a \phi^a \bar{H}_3 \bar{F}_3 \\ & + \; T_3 \phi^a \bar{H}_a \bar{F}_3 + T_a (S^{ab} + A^{ab}) \bar{H}_3 \bar{F}_b + T_a (S^{ab} + A^{ab}) \bar{H}_b \bar{F}_3 + T_3 (S^{ab} + A^{ab}) \bar{H}_a \bar{F}_b \Big)\, \xi \Big] \\ & + \frac{1}{M_f^2} \Big[T_a (S^{ab}\Sigma + A^{ab}\Sigma + \phi^a \phi^b) H_3 T_b + T_3 (S^{ab}\Sigma + A^{ab}\Sigma + \phi^a \phi^b) H_a T_b \\ & + \Big(T_3 \bar{H}_a \phi^a \phi^b \bar{F}_b + T_a \phi^a \phi^b \bar{H}_b \bar{F}_3 + T_a (S^{ab} + A^{ab}) \phi^c \bar{H}_b \bar{F}_c \Big)\, \xi \Big] \\ & + \frac{1}{M_f^3} \Big[T_a \phi^a \phi^b \phi^c H_b T_c + T_a \phi^a \phi^b \phi^c \bar{H}_b \bar{F}_c\, \xi \Big] \,, \end{aligned} \tag{5.5}$$

where $M_f \equiv \epsilon M_{\mathrm{GUT}}$ is the flavor scale and $\xi \sim m_b/m_t$.

The Yukawa coupling matrices can now be obtained from Eq. (5.5) and Eq. (5.4), their textures are given by

$$Y_U \approx \begin{pmatrix} 0 & \epsilon\epsilon' & 0 \\ -\epsilon\epsilon' & \epsilon^2 & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} H_3 + \begin{pmatrix} 0 & \epsilon\epsilon' & \epsilon\epsilon' \\ -\epsilon\epsilon' & \epsilon^3 & \epsilon^2 \\ -\epsilon\epsilon' & \epsilon^2 & \epsilon \end{pmatrix} H_2 + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon\epsilon' \\ 0 & \epsilon\epsilon' & 0 \end{pmatrix} H_1 \,, \tag{5.6}$$

$$Y_D \approx \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} \xi \bar{H}_3 + \begin{pmatrix} 0 & \epsilon\epsilon' & \epsilon' \\ -\epsilon\epsilon' & \epsilon^2 & \epsilon \\ -\epsilon' & \epsilon & \epsilon \end{pmatrix} \xi \bar{H}_2 + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\epsilon\epsilon' & -\epsilon' \\ 0 & \epsilon' & 0 \end{pmatrix} \xi \bar{H}_1 \,, \tag{5.7}$$

where $O(1)$ coefficients have been omitted. Using the values $\epsilon \approx 0.02$ and $\epsilon' \approx 0.004$ obtained from fitting the quark masses and mixing angles [6], one can calculate the contribution to FCNC.

A comprehensive analysis of flavor-changing processes in models with tree-level FCNC was performed in Ref. [8]. If one assumes that the flavor-changing coupling $q_i q_j \phi$ is the geometric mean of the q_i and q_j Yukawa couplings, then the strongest bound comes from $K^0 - \bar{K}^0$ mixing; the mass of the exchanged scalar had to be greater than approximately a TeV. Much weaker bounds came from processes involving heavier quarks. In the model of this Section the Yukawa couplings are different; the $d\,s\,H_2$ coupling is $O(\epsilon\epsilon'\xi)$ which is $\sqrt{\epsilon'}$ times the geometric mean of the down and strange Yukawa couplings. This significantly weakens the bound; we find a bound on the H_2 mass of 17 GeV. in the case of $B^0 - \bar{B}^0$ mixing we obtain a bound of 100 GeV (with $f_B \approx 200$ MeV). The bound coming from $D^0 - \bar{D}^0$ mixing is of ~ 120 GeV. Bounds on processes involving leptons and b-quarks are much weaker. Since $D^0 - \bar{D}^0$ mixing is negligible in the standard model, the first signature of this model could come from $D^0 - \bar{D}^0$ mixing.

A similar analysis to the one described in the previous sections could now be made. In this case however, the "extra" Higgs bosons do acquire vevs and couple to fermions, as can be seen from Eqs. (5.6−5.7). Thus, they all decay at the vertex. We do not perform a detailed study of the phenomenology of the additional Higgs scalars.

VI. CONCLUSION

We study the supersymmetric standard model with more than one generation of Higgs doublets. We follow Ref. [4] where a symmetry that forbids tree-level FCNC has been assumed. A result of this assumption is that the additional Higgs bosons decouple from the standard Higgs bosons, do not couple to fermions, and that there is a mass degeneracy among the lightest neutral bosons. Radiative corrections lift this degeneracy, and thus provide interesting phenomenological considerations. In this paper, the mass splittings are calculated and it is found that the neutral pseudoscalar is heavier than the neutral scalar by $O(0.5 - 1.5)$ MeV, and that the charged scalar is heavier than both by several GeV. The decay modes of the pseudoscalar and charged scalar are also analyzed. We find that standard searches for heavy leptons with neutrinos a few GeV lighter will be sensitive to these particles, and an additional signature coming from the decay of the charged scalar to the pseudoscalar will be present in over half the events. Lastly, we relax the assumption that tree-level FCNC are eliminated, and discuss a flavor symmetry based on U(2) showing how the level of FCNC processes can be related to quark masses.

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